Challenges in Theoretical Physics

$$V(R) = -G_N \frac{m_1 m_2}{R}$$

$$V(R) = \frac{q_1 q_2}{R}$$

singular at $R=0$!

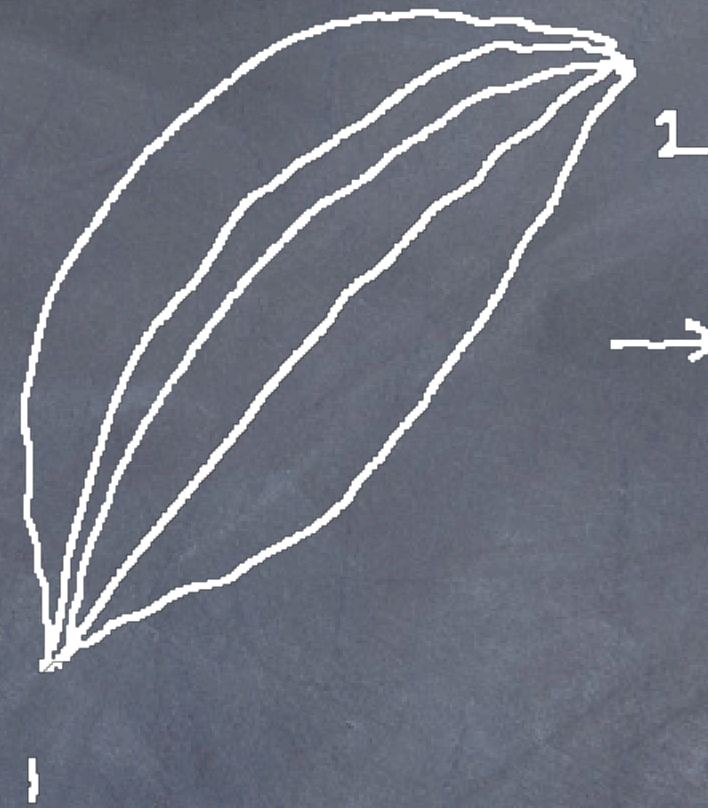
- Planetary motion
- ? stability of atoms ?

Classical Physics

Quantum Mechanics

$$\Delta x \, \Delta p \geq \hbar/2$$

Trajectory not meaningful, at best "fuzzy"



→ e^- does not "find" p !

Energy of radiation now depends on wavelength → NO UV CATASTROPHE IN BLACKBODY RADIATION

Quantum Mechanics + Special Relativity = Quantum Field Theory

At each point in space-time there are quantum degrees of freedom interacting with their nearest neighbor

What can we calculate?

What are meaningful questions?

$$L = \int d^3x \left(\frac{\bar{E}^2}{2} - \frac{\bar{B}^2}{2} + \underline{c_1} (\bar{E}.\bar{B})^2 + \underline{c_2} (\bar{E}^2 - \bar{B}^2)^2 + \cdots \right)$$

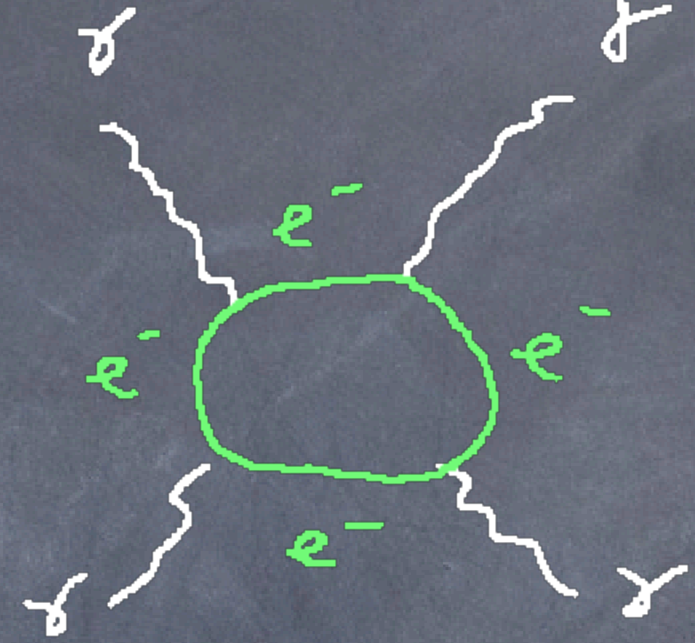
Scattering of photons at $\omega << 1\ \mathrm{MeV}$

c_1 and c_2 to be measured

$C_i \qquad\qquad \sim c_i c_j \Lambda_{UV}^4$

$\gamma + \gamma \rightarrow \gamma + \gamma$ ULTRA SENSITIVE TO SHORT DISTANCE PHYSICS

$E > 1\ \text{MeV}$



$$C_1 = \frac{14\alpha^2}{45 m_e^4}$$

$$C_2 = \frac{C_1}{7}$$

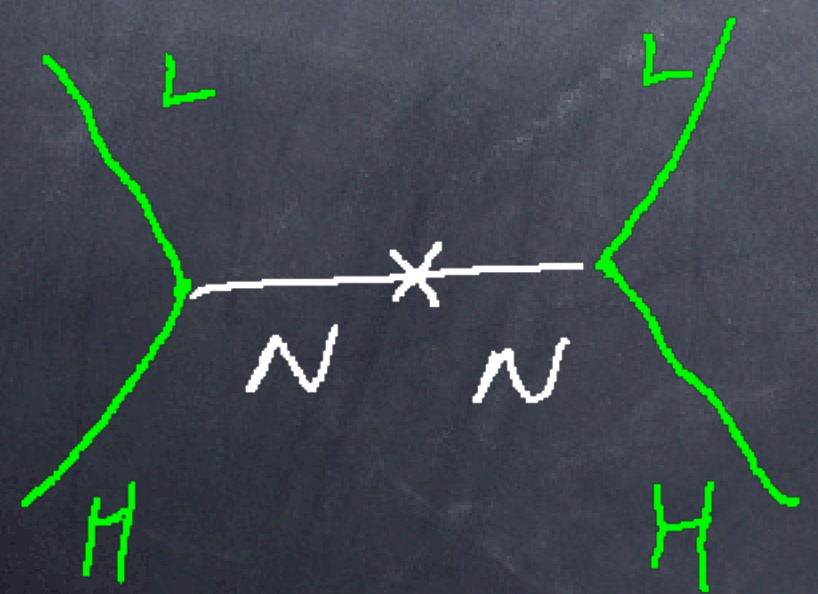
New degrees of freedom: massive charged particles

footnote: massive neutrinos

STANDARD OF ELECTRO-WEAK AND STRONG INTERACTIONS $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\delta L = \frac{\left(L^{\alpha}_{(-1/2)} H^{(+1/2)}_{\alpha}\right)^2}{M} = \frac{\left(\nu \langle H \rangle\right)^2}{M} = \frac{\nu\nu}{M G_F}$$

$$m_\nu = \frac{1}{M G_F} \sim 10^{-2}\,eV \rightarrow M \sim 10^{16}\,GeV$$

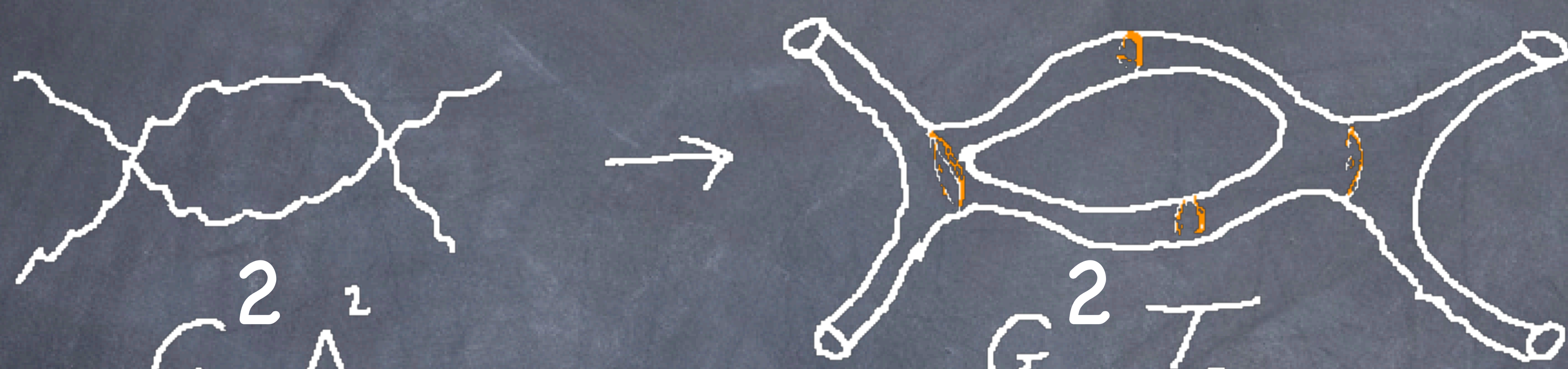


New degree of freedom
10^{16} GeV SINGLET neutrino

graviton - graviton scattering

WHAT ARE THE NEW HIGH ENERGY DEGREES OF FREEDOM ?

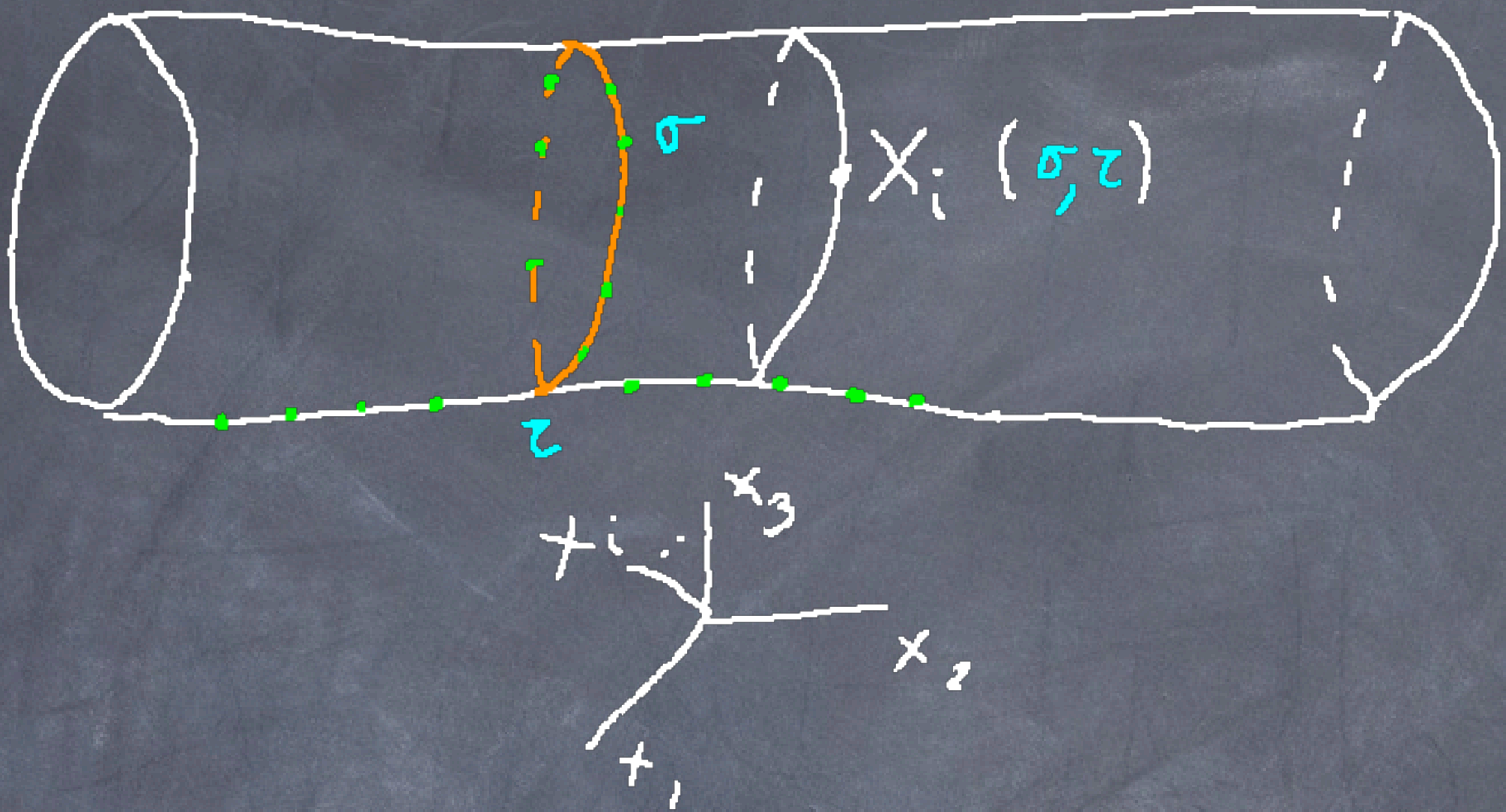
ANSWER: INCLUDE ALL THE MODES OF A RELATIVISTIC STRING!



$G_N^2 \Lambda^2$ $\qquad$ $G_N^2 T_s$

$m_s \sim \sqrt{T_s}$

SHORT DISTANCE PHYSICS SOFTENED BY EXCITING MODES OF THE STRING



$$\left\langle \left(X_i(\sigma,\tau) - \int d\sigma\, X_i(\sigma,\tau) \right)^2 \right\rangle \sim \log r$$

r : short distance resolution on the string

BLACK HOLES

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 d\Omega$$

$r_H = 2M$: event horizon

$M \nearrow \longrightarrow r_H \nearrow$ The bigger the energy, the larger the black hole!

$\longrightarrow$ gravity limits probing short distances.

Star filled with radiation ($p = \frac{1}{3}\rho$)

$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + r^2 d\Omega$$

TOV EQUATIONS:

$$\frac{1}{r}\frac{\partial_r (AB)}{AB^2} = 8\pi(p+\rho)$$

$$\frac{2}{r^2}\left(1 + \frac{1}{B} - \partial_r\left(\frac{r}{B}\right)\right) = 16\pi\rho$$

$$\partial_r p = -(p+\rho)\frac{\partial_r A}{2A}$$

Equations are invariant for:

$$r \to \lambda r, \quad p \to \frac{1}{\lambda^2} p, \quad \rho \to \frac{1}{\lambda^2}\rho$$

$$M \sim \int_0^R dr\, r^2 \rho(r) \sim R$$

$$S \sim \int_0^R dr\, r^2 \sqrt{B(r)}\, \sigma(r)$$

$$\uparrow$$

$$\rho^{3/4}$$

$$S \sim R^{3/2} \;!$$

Threshold of black hole formation:

$\forall\, \bar{k}$, $\varepsilon_{\bar{k}} = |\bar{k}|$ in box of linear size $|\bar{k}|$

$$\rightarrow \rho \sim \frac{1}{|\bar{k}|^2} \rightarrow \sigma \sim \frac{1}{|\bar{k}|^{3/2}}$$

$$S \sim \int_0^R d|\bar{k}|\, |\bar{k}|^2 \frac{1}{|\bar{k}|^{3/2}} \sim R^{3/2}$$

Black hole of size R and mass R:

Entropy $S = \frac{4\pi R^2}{4}$, way more efficient in packing entropy than any other form of matter!

In the presence of gravity, S extensive with Area NOT VOLUME!

The Temperature of the black hole

$$T = \frac{1}{R} \longrightarrow \text{STEFAN'S LAW}$$

$$\frac{dM}{dt} = \text{AREA} \times T^4$$

$$\frac{dM}{dt} \sim \frac{M^2}{M^4} \sim \frac{1}{M^2}$$

Lifetime of Black Hole $\sim M^3$

A SOLAR MASS BH:

$R \sim 10^5$ cm , $S \sim 10^{76}$

$\tau \sim 10^{64}$ yrs $T \sim 10^{-9}$ eV

Is quantum mechanics in the presence of black holes unitary?

The far away observer has no access to the events beyond the horizon.

The infalling observer has no way to detect the horizon. He is seen from far away as eventually burning up near the horizon.

Holography, Complementarity

The challenge in bringing together quantum mechanics and gravity:

In quantum mechanics in the absence of gravity, one uses "heavy" systems, to be used as classical measuring devices, "light" ones would fluctuate quantum mechanically.

On the other hand in classical gravity, when measuring the geometry of space-time one uses light rods and clocks such as to minimize the influence on the geometry.

What do we do when we put the two together?

If space-time is asymptotically flat, one can define a mathematically precise quantity:

The S-matrix

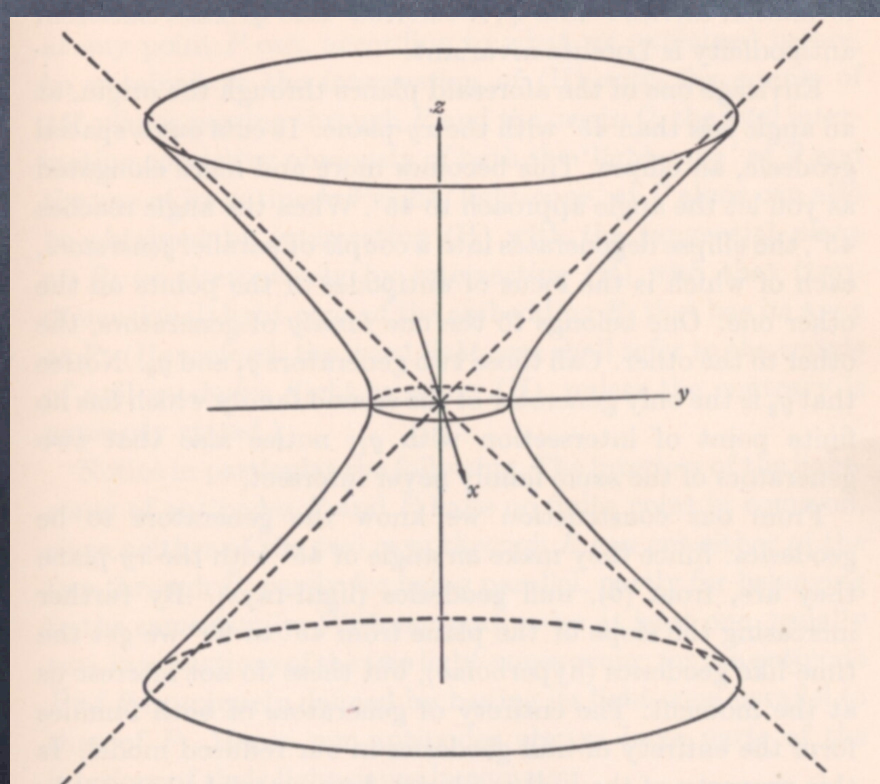
Our universe is accelerating!

No S-matrix!

Assume the acceleration of the universe is due to a cosmological constant: $\Lambda \sim (10^{-3}\,\mathrm{eV})^4$

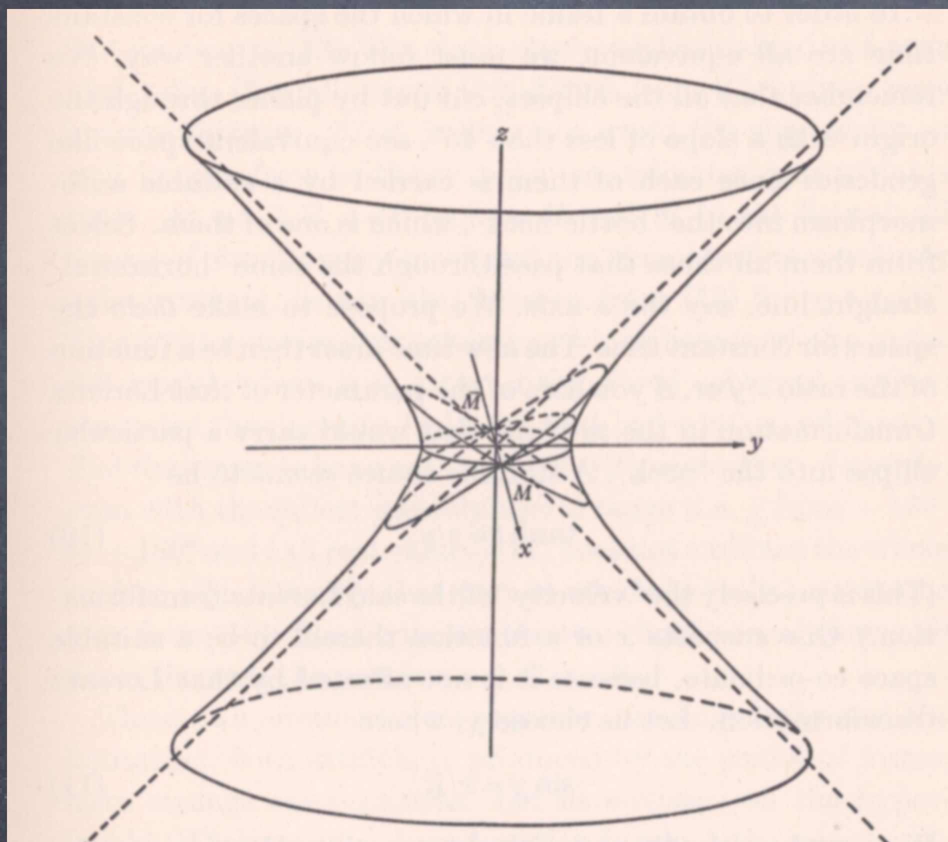
Our universe becomes asymptotically

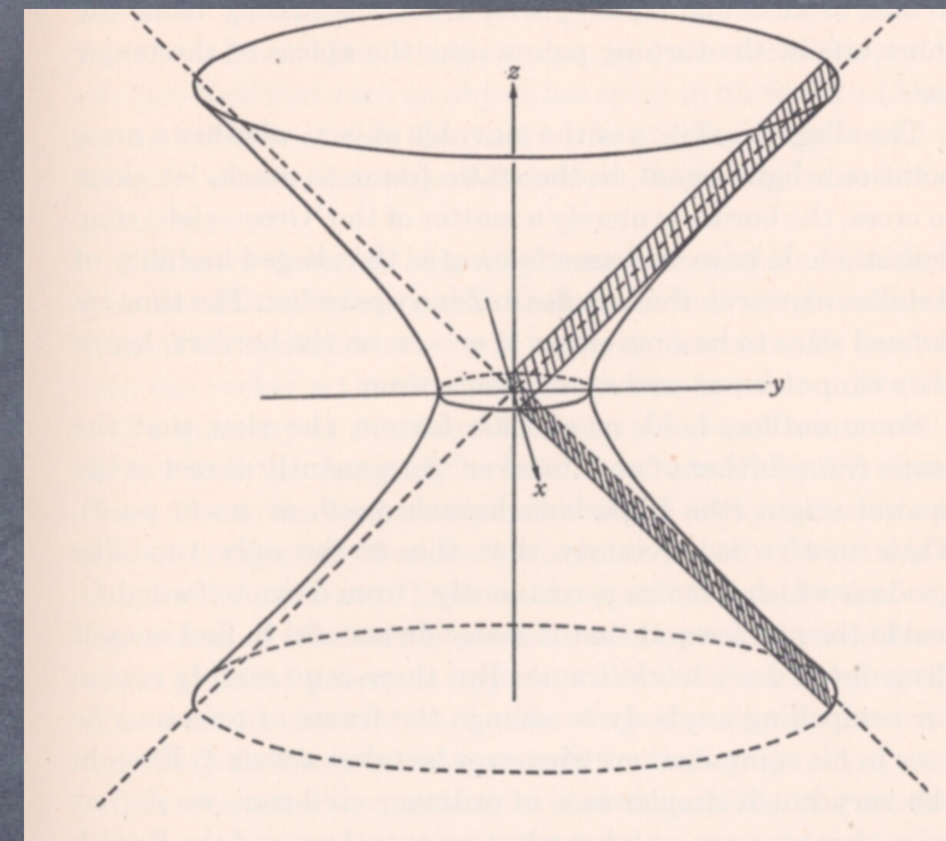
"de Sitter space-time"



$$x^2 + y^2 + v^2 + u^2 - z^2 = R^2$$

Onto the Static Frame



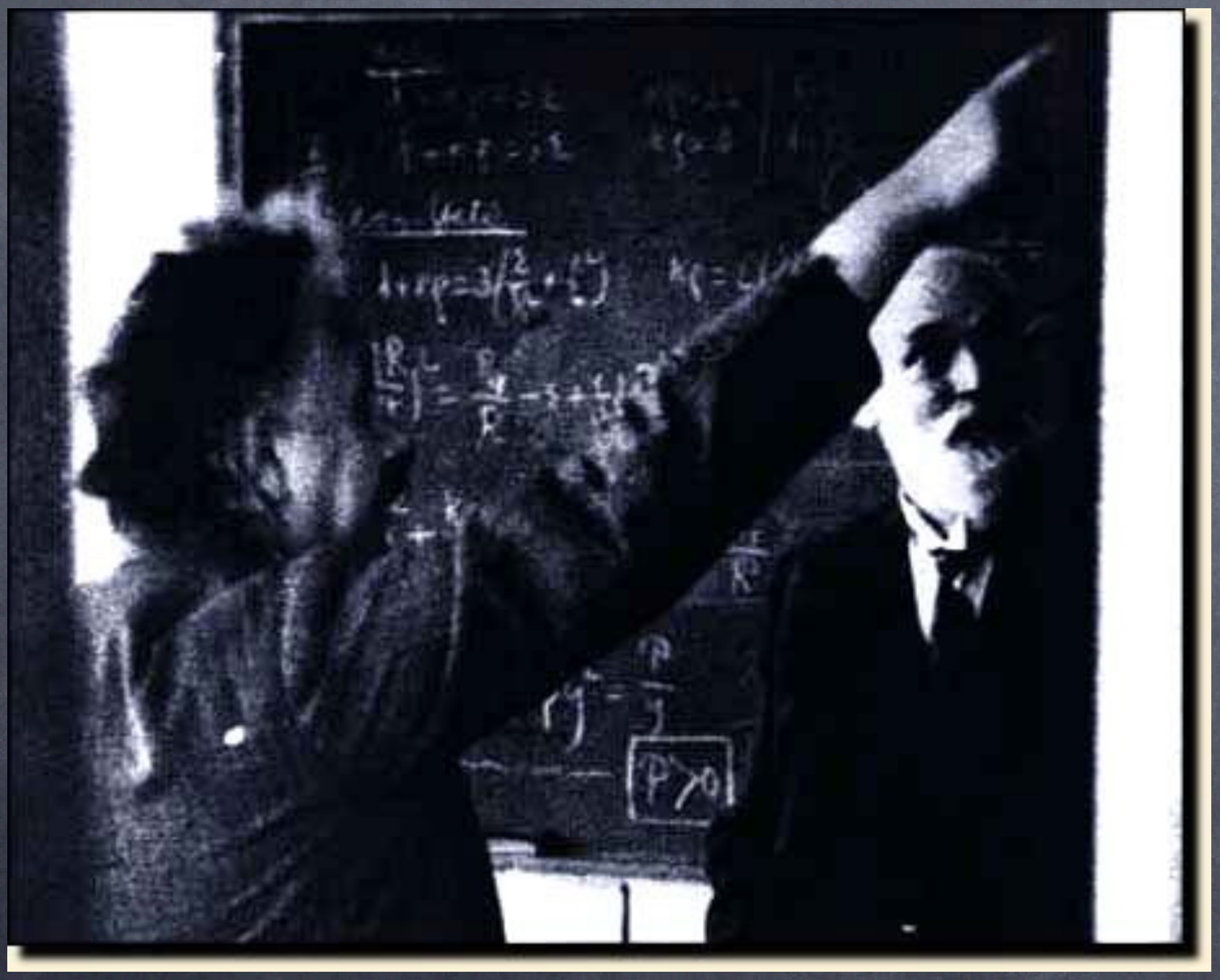


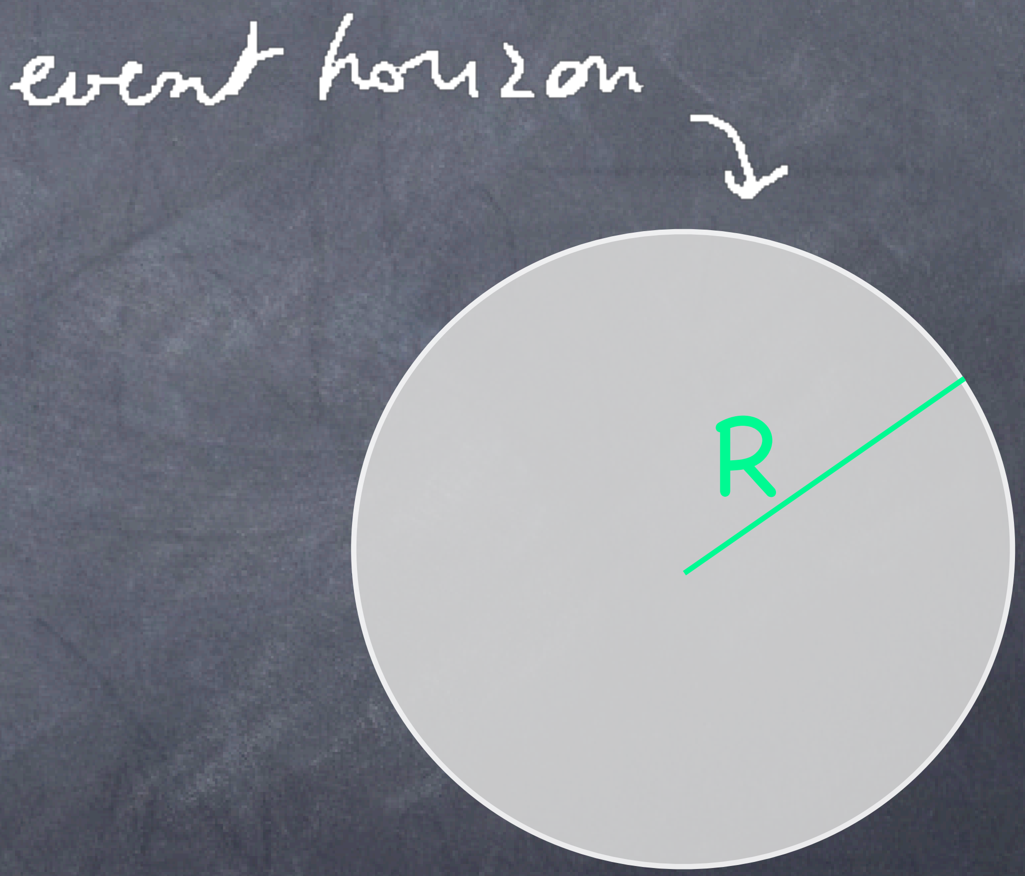
$$\operatorname{th}\frac{t}{R} = \frac{z}{y} \; ; \quad x = R\sin\theta$$

$$y = R\cos\theta\cosh t/R$$

$$z = R\cos\theta\,\operatorname{sh} t/R$$

$$ds^2 = \left(1 - \frac{r^2}{R^2}\right) dt^2 - \frac{dr^2}{1 - \frac{r^2}{R^2}} - r^2 d\Omega$$





Black hole in de Sitter space

$$ds^2 = \left(1 - \frac{2M}{r} - \frac{r^2}{R^2}\right) dt^2 - \frac{dr^2}{1 - 2\frac{M}{r} - \frac{r^2}{R^2}} - r^2 d\Omega$$

empty de Sitter space

- Local observer sees a finite amount of entropy: $S = \frac{\text{Area}}{4} = \frac{4\pi R^2}{4}$

$= \frac{\pi}{3\Lambda}$

- $\exists$ maximum size black hole
  - → UV cut-off on mass
- energy levels discrete
  - → FINITE DIMENSIONAL HILBERT SPACE

- Limitation on measurements!

- What is the quantum theory of asymptotic de Sitter space-time?

- Is there such a theory?

- What does this mean for String Theory?

Stay tuned...

“And, as imagination bodies forth
The forms of things unknown, the poet's pen
Turns them into shapes, and gives to airy nothing
A local habitation and a name”

– William Shakespeare, A Midsummer Night's Dream